

# OPTIMIZATION OF THE MINIMUM COST/ MAXIMUM YIELD RATIO IN THE GROUNDWATER MANAGEMENT: PROPOSAL OF AN OPERATIVE METHOD FOR LOMBARDY

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## **RIASSUNTO**

La pianificazione delle reti acquedottistiche è un problema assai complesso in quanto oltre a richiedere la conoscenza di numerosi fattori di carattere prevalentemente idrogeologico, necessita di investimenti molto elevati. Per tale motivo appare necessario esaminare gli aspetti scientifici ed economici che costituiscono gli elementi basilari delle linee guida per la gestione delle acque sotterranee.

Questo obiettivo è stato raggiunto utilizzando l'accoppiamento di modelli di simulazione e di ottimizzazione prendendo come esempio di analisi il territorio della pianura di Milano, sul quale da tempo sono in corso studi sperimentale di questo tipo. In particolare, il modello di ottimizzazione è basato su un metodo di programmazione lineare che consente di identificare le aree delle quali si possono estrarre con il minimo costo e con la maggiore resa le quantità d'acqua necessarie per fronteggiare la domanda prevista. Tale impostazione permette di procedere nel caso della pianura milanese ad una gestione idonea ad affrontare i suoi grandi problemi, in quanto obbliga a concentrare una buona parte dei prelievi sulle acque che, ove non captate, sarebbero destinate comunque a non venire utilizzate dal sistema o a peggiorare la propria qualità al punto da superare i livelli di contaminazione ammissibili per la potabilità.

Si ritiene che l'adozione di tale metodologia consenta la realizzazione di piani acquedottistici soddisfacenti anche per le altre province della pianura lombarda.

**Parole chiave:** gestione acque sotterranee, modello idrogeologico, costi, pozzi.

## **ABSTRACT**

In this paper the main items of the aqueduct planning are explained. Particularly, this planning is based on two models: an hydrogeological model ( Heidari's model) and a cost optimization model. By means of these two models it is possible to achieve a rational choice of the areas that are able to satisfy the water demand. The territory of Milano province has been selected as case history.

**Key words:** Groundwater management, Aqueduct Planning, Hydrogeological model, cost estimation, bearing wells location.

## **1. INTRODUCTION**

The law prescriptions concerning the groundwater pollution forced the Italian Regional Governments to introduce the aqueducts planning. This planning needs to be carried out in a suitable way for two reason. On the one hand, the aqueduct planning conditions the urban and industrial growth, on the other it requires very high investments. For this reason, it is very important to deal with the aqueduct planning and the groundwater management examining the

scientific and economical aspects on the ground of experience achieved in Europe, USA and Canada.

Particularly, in this paper the main items of aqueduct planning are explained. This planning is based on two jointed models: an hydrogeological model and a cost optimization model.

By means of these two models it is possible to achieve a rational choice of the areas able to satisfy the future water demand ( forecasted by means of an appropriate simulation algorithm) under the constraint- objective of the minimum cost/ maximum yield ratio optimization. The territory of Milano Province has been selected as case history. Several studies in fact on the groundwater management of this zone have been performed.

## **2. THE BASIS OF THE AQUEDUCT PLANNING**

In order to carry out correctly a planning of the aqueducts, a preliminary knowledge of groundwater resources and their relative location is required. These information can be obtained by the hydrogeological cartography. This latter can be easily realized: the available data about the underground features and the groundwater quality are reliable and numerous.

The present requirement of irriguous, industrial and potable water needs to be quantified. Then, it is necessary to estimate the future water requirements on the ground of the demographic increase and of the new agricultural and industrial installations.

Furthermore, the forecast of the trend of the groundwater balance in the time is required. This items needs to consider both the water demand increase and the change of groundwater input output conditions. In this latter phase, it is important to evaluate the fluctuations of the piezometric head which regards the changes of groundwater resource. In this way, the behavior of the piezometric head to changes of the groundwater input/output conditions can be displayed.

In this phase of aqueduct planning, the employment of an hydrogeological model is required. This model allows to make a rational and optimal choice of the areas where the drawings could be increased and, at the same time, to estimate the entity of these drawings on the basis of the future water demand.

The hydrogeological model must be coupled with a cost optimization model in order to define, from an economical point of view, the most convenient solutions among the alternatives selected by the hydrogeological model.

Before describing the contents and the operative modalities of the hydrogeological model, it seems necessary to explain the meaning of the management of groundwater estate taking into particular consideration the significant variables in order to optimize the use of the groundwater estate.

### **3. THE MANAGEMENT OF THE GROUNDWATER ESTATE**

In this paper the groundwater estate is considered in the economical meaning so that methods of the groundwater management can better evaluated.

Considering the groundwater resource as an estate allows to value the economical loss connected to a sub-optimal management, evaluating, at the same time, the benefits of a rational exploitation.

The groundwater estate of a area includes the existing hydrogeological reserve plus the amount of water stored or lost during the hydrogeological cycle. The stored water depends mainly on rainfall, river contributions and infiltration due to the irrigation. On the contrary, the amount of the lost water depends on drainage of the rivers, stratum output flow, drawings and evapotranspirations.

Normally, the hydrogeological balance is in equilibrium, i.e. the amount of stored water is equal to the amount of lost water.

In reality, the balance could be negative. It happens, for example, in some Provinces of Lombardy at the end of 80s.

This facts products a piezometric depression making evident the progressive decrease of the volume of the hydrogeological reserve.

To avoid this decrease is necessary to verify the opportunity to change some components of the groundwater balance. In order to apport some change to balance, from an optimization point of view, it is possible to decrease the groundwater pollution characterizing a relevant part of North Italian groundwater and to optimize the location of the sites of hydrogeological reserves exploitation.

The increase of the water availability, by means of decrease of pollution, is only feasible only by applying new laws at present in force in Italy, and from a longtime in use in other European countries as Swiss, Great Britain, Spain etc. these latter in fact issue laws about the safeguard of the areas where wells and springs are present.

The only way to modify the groundwater balance seems to achieve a more rational location of the water well disposal. In fact, the actual deficit of groundwater balance is mainly depending on the wells location near or into cities, town an factories. This location has been justified by the Government Authorities on the basis of the apparently lower cost.

In reality, this location choice induces a strong increase of the pollution and a growing piezometric depression with a consequent progressive decrease of hydrogeological reserve. In many European countries, the techniques of optimal wells allocation and of groundwater policy evaluation are well developed: for this reason a preliminary explanation of the main contents of the existing techniques is required.

#### **4. THE HYDROGEOLOGICAL MODEL: THE CONCEPT OF LOCAL HYDROGEOLOGICAL RESERVE (LHR)**

Many authors have introduced the term *Hydrosystem* to indicate the whole complex of interconnected surface and sub-surface water. Within the hydrosystem an *hydrologic system* and an *hydrogeologic system* are distinguishable.

In fact, the surface and subsurface water present generally a different behaviour caused by different components.

Moreover, an hydrogeological system can be subdivided in several parts, everyone consisting of a subsystem which generally coincides with an hydrogeological structure. If the outlines of these structures are conventional or arbitrary fixed, in many cases, they coincide with exchange limits. We define the reserve stored in such structures as Local Hydrogeological Reserve (LHR). The complex of LHR forms the global reserve of the hydrosystem.

#### **5. THE HEIDARI'S MODEL**

The hydrogeological model, proposed by Heidari (1982), premise to optimize the location of water wells and to determine the amount of discharge in relation to the drawdown of the piezometric head.

The Heidari's model is founded on the transformation of the state  $S$  of the hydrogeological system for naturally water demand into a desired state  $S^*$ .  $S^*$  is a function of the desired quantity  $V^*$  and quality  $Q^*$  both depending by desired time  $T^*$  and the desired location  $X^*$ .

The desired state  $S^*$  can then be expressed as:

$$S^* = [V^*(X^*, T^*), Q^*(X^*, T^*)] \quad (1)$$

In order to research  $S^*$ , many authors have pointed out the solution of the response matrix approach, using a groundwater simulation model external to the optimization model.

An unit response describes the influence of a pulse stimulus (such as a unit pumpage or injection over a time period) at a selected location (well) or cell upon the hydraulic heads within the modelled area.

The response matrix consists of all the unit responses. Maddock (1972) derived a function relating the drawdown in a confined aquifer to pumpage through the use of unity response function

this function, also referred as an algebraic technological function, defines the drawdown  $s(k,n)$  in the  $k^{\text{th}}$  cell at the end of the  $n^{\text{th}}$  time period.

$$S(k, n) = \sum_{p=1}^n \sum_{j=1}^J \beta(k, j, n, p) q(j, p) \quad (2)$$

Where:

- the unit response function  $\beta$  is the change of drawdown (unit drawdown) in the  $k^{\text{th}}$  cell at the end of the  $n^{\text{th}}$  time period due to an unit pumpage from the  $j^{\text{th}}$  cell during the  $p^{\text{th}}$  time period;
- $Q(j, p)$  is the quantity pumped from  $j$  during the  $p^{\text{th}}$  time period;
- $J$  is the number of cells.

At the 1982, Heidari formulated a groundwater management model, based on the response function approach as follows:

$$\text{Maximize } z = \sum_{j=1}^J \sum_{n=1}^n q(j, n) \quad (3)$$

Subject to:

- a. Satisfying the governing equation of flow through the response matrix;
- b. Pumpage cannot exceed  $q(j, n)$  which is the smaller of the appropriated right or its capacity;
- c. Drawdown at each well or cell cannot exceed an upper limit  $s(k, n)$
- d. Demand  $Q$  should be satisfied for each period of time.

The upper limit  $s(k, n)$  can be defined as a fraction  $\phi$  of the saturated thickness  $b(k)$  of the aquifer at the  $k^{\text{th}}$  cell so that:

$$S(k, n) = \phi b(k) \quad (4)$$

## 6. OPERATIVE METHOD FOR LOMBARDY

### 6.1. HYDROGEOLOGICAL FEATURES OF THE LOMBARDY

The balance of the groundwater in Milano Province, performed in 1985 by section of applied Geology of Politecnico di Milano, demonstrates that the alimentation of hydrogeological reserves deriving from groundwater is about 50 mc/s, while that deriving from irrigation recharge by rain water infiltration is about 60 mc/s.

The hydrogeological reserve is increased also by about 3 mc/s of water deriving from sewage plants or recycling operations. Therefore, a supply of about 50 mc/s, consisting of 25 mc/s for industrial purposed and 25 mc/s for urban uses, is derived from the hydrogeological reserve.

About 20 mc/s are the irrecoverable losses due to evapotranspiration or industrial operations (chemical compounds and other products).

The rivers and the streams receive the major part of wastes deriving from sewage plants. The amount of these affluxes is about 70 mc/s, partly consisting of groundwater discharge (we have estimated a value of 10 mc/s).

A part of effluents from industrial plants (about 5 mc/s) consists of wastes representing the main cause of the growing pollution which strongly reduces the water reserve.

The provinces of Lodi and Pavia are reached by the remaining part of the discharge of the groundwater, i.e. about 30 mc/s.

These values of the balance was negative are referred to an ideal year in which the affluxes are equal to the defluxes.

In the 80s the groundwater balance was negative with a deficit of about 15 millions of mc/y. this caused an average decrease of groundwater head of 0.5m.

For this reason, in prospective of a future growth of the water demand, a strong correction of the groundwater management seems unavoidable. In order to determine the methods for reaching this objective and for estimating the maxima quantities of groundwater exploitables without damage of the hydrogeological equilibrium, we have performed the elaborations exposed in the following chapter.

## **6.2. SOME ADJUSTMENTS OF HEIDARI'S MODEL**

In the preceding paragraph we have described the particular problems of the Lombardy. Particularly the large distance between the sites characterized by high hydrogeological productivity and the most important towns imply huge costs for the aqueducts. Moreover the extraction activity of groundwater determines marked decrease of hydrogeological reserve.

The hydrogeological Heidari's model allows the optimization of the groundwater managements by means of the minimization of the drawdown of the piezometric head caused by the discharge of wells.

This minimization can be obtained fixing a lower limit to the piezometric depression in correspondence of the most important cities. This lower limit corresponds to 1m below the average piezometric head registered in 1992.

The Heidari's model allows to calculate the discharge of wells corresponding to the fixed limit of drawdown in the cells representing the most productive structures.

The final result of these studies indicates in 5 mc/s the amount of water that can be derived without damage for the hydrogeological reserve.

At the end, for each cell, corresponding to local hydrogeological structure, the drawdown and the decrease of the LHR have been calculated.

Summarizing the application of Heidari's model permits to highlight that the most important intervention on the groundwater balance consists in the utilization of water going off the hydrosystem and reaching the river Ticino or river Adda ( for the Milano province about 10 mc/s).

In fact at present this water having a good quality is quickly polluted by wastes and sludges collected by watercourses; hence its utilization for aqueducts purposed is impossible.

For this reason we can suppose that there aren't any LHR losses caused by the exit of groundwater supplying the watercourses.

Therefore in the cells located near limits with imposed potential (representing the rivers which receive groundwater) the LHR is particularly high; in fact the whole groundwater going out the structure is lost for the hydrogeological balance.

Consequently the employ cost of this water is very low and the hydrogeological balance is not worsted by groundwater withdrawal in these structures. On the contrary, the hydrogeological reserves of the central part of the Milano province are historically overexploited.

The supply deriving in this central part from infiltration of meteoric and irrigue water is very poor and the groundwater level is rapidly decreasing. On the other hand the percentage of polluted water is in this region very high.

Hence the increasing development of piezometric depression area, consequent to the deficit of the hydrogeological balance, gives rise to a converging flux of contaminant toward the major cities.

It is therefore necessary introduce in the model these contextual data highlighting the minimal loss of LHR near the watercourses and the increase of contaminants afflux toward the water bearing wells. Finally the modified hydrogeological model allows to choose the areas where water demand should be satisfied without danger for the hydrogeological reserves.

However the above mentioned hydrogeological model doesn't consider the economical aspects holding on the contrary a great importance in the groundwater management. For this reason the Heidari's model must be supported by an optimization cost model able to choose the best solution from both an hydrogeological and economical point of view.

## **7. THE COST OPTIMIZATION MODEL**

The hydrogeological solution derived from the employment of Heidari's model require different investments. Consequently the investments of each solution must be evaluated. Preliminarily the



drawdown, construction, maintenance quality control and management costs have been calculate for each cell.

By multiplying the matrices of costs for the drawdown in each cell, the cost for each solution deriving from the piezometric depression can be obtained.

Finally, adding the construction, maintenance, qualitative control and management costs of the whole aqueducts, we determine the global costs of each solution.

On this latter point, a synthetic description seems useful.

The calculation of global cost associated to a selected solution is based on a system of equation describing the behavior of the global cost of building (CBB) with respect to different factors.

The expenses for labour and operators (OC) and for plants maintenance (MC), are added to the global cost of building to obtain the total structural cost (TSC).

The TSC plus the expenses of decontamination (DC), electrical energy (EC) and water consumption due to withdrawl (WCW) give the total cost (TC).

Finally the cost of water consumption is determined from the piezometric depression calculated by means of the hydrogeological model.

Now, if X indicate the discharge, we have for each solution:

$$TC = TSC + X(WCW + DC + EC) = (CBB + OC + MC) + X(WCW + DC + EC) \quad (5)$$

Moreover, if PVC represents the cost at time n (forecast), we have too:

$$PVC = TC + TC'/(1+i) + TC''/(1+i)^2 + TC^n/(1+i)^n \quad (6)$$

Where the cost components depending of discharge are EC, DC, WCW. Therefore, for the generic solution, we can write:

$$TC1 = TS1 + X (DC1 + EC1 + WCW1) \quad (7)$$

$$PVC1 = TC1 + TC1'/(1+i) + TC1''/(1+i)^2 + TC1^n/(1+i)^n \quad (8)$$

Where DC, EC, WCW are the specific cost for unity of discharge. The above exposed system of equations represents the basis to define a cost optimization model adaptable to the Heidary's model. The starting assumption is that the cost optimization depends strongly from the discharge X.

However, it is necessary to calculate preliminarily the solution for minimum drawdown using the Heidary's method.

Only at this point, it is possible to solve the cost optimization problem, introducing both the values of optimum discharge X and the location of water bearing wells in the cost optimization model.

In practice, we build a system of equation, solving for PVC:

$$PVC1 = TSC1 + f1(q)$$

$$PVC2 = TSC2 + f2(q)$$

$$PVC3 = TSC3 + f3(q)$$

$$PVCn = TSCn + fn(q)$$

Where PVC 1, 2, ...n correspond to the solutions that, on the basis of Heidari's model, determine the minimum  $\delta LHR$ .

Each solution induces different EC, DC and TSC. Hence, the application of a LP model to solve this allocation problem is necessary.

For this reason the relation between EC, DC, WCW and q has been examined considering the criteria adopted by the Regione Lombardia to plan the management of water resources (1992).

By applying this criteria, we have determined for each area the costs of groundwater exploitation (TSC).

## **8. ANALYSIS OF THE COMPONENTS TOTAL COST**

The cost of the groundwater changes in the different towns of Lombardy according to the local demand; for example, the cost of one mc of water in several town is on average equal to 0.60 € (it was about 0.10 € in 1993) for a demand of about 100-200 l/day, while in presence of a demand of about 1000mc/day the cost rises until 0.90 € (0.26€ in 1993).

However, in many towns the above mentioned criteria are not completely respected.

Consequently, we can attribute to the groundwater a cost of 0.60 - 0.70 €/mc (0.1 €/mc in 1993) , corresponding with reasonable approximation to the cost of water trading. This cost represents approximation the WCW.

The cost of aqueducts maintenance are formed by:

- workmen and operators
- Energy
- Analytical controls
- Maintenance
- General expenses

The cost of energy depends on q and it can be determined by means of the following relationship:

$$Ec = 0.60€ \cdot HV / (367) \quad (10)$$

The other factors of aqueducts maintenance have been evaluated by the Regione Lombardia using similar relation.

The decontamination cost (DC) depends on the peculiar treatment of potabilisation conducted.

The Regione Lombardia define six different typologies of treatment synthesized in table 1.

Table of adjunctive costs of potabilisation plants	
Typology 1	Water of river and artificial channels
Typology 2	Treatment by means of oxidations of Fe, Mn, H <sub>2</sub> S
Typology 3	Treatment of purification from organic micropollution by means of filtering on active charbon
Typology 4	Biological removal of nitrates
Typology 5	Oxygenation with removal of H <sub>2</sub> S, CH <sub>4</sub> , NH <sub>3</sub> and organic compounds
Typology 6	Removal of sulfates by means of electro dialysis

Considering only the province of Milano (it is the least expensive province in Lombardy), the treatment of potabilization more diffused are those corresponding to the typologies 4 and 5. The average cost (DC) is therefore equal to 0.20 €/mc.

The cost depending on construction and maintenance (inclusive of the analytical controls, monitoring, general expenses), changes according to the length of the aqueducts ( from 1032.91 €/km for very short aqueducts to about 516.46 €/km for more than 10 km of length).

The latter consideration allows to see the structural cost increases slowly to the change of the distance from the water bearing wells, whilst it sharply increases to the change of the discharge and contamination.

## **9. DETERMINATION OF OPTIONAL LOCATION OF WATER BEARING WELLS**

We see that the optimal solution requires above all the minimization of decontamination and withdrawals costs, which strongly affects the total costs (TC). This requires to carefully consider the problem of the location of the water bearing wells.

In any case, in the Milano province the number of suitable solutions is very low: for this reason the solution with the minor cost present the following requirements:

- The location of new wells is always nearby the rivers,
- A noticeable contribution is required to superficial water (namely to lakes),

- The aqueducts working at present must be maintained,
- Frequent control and monitoring of the water quality are required to maintain a good quality.

In fact applying the Heidari's model, we can define a very large set of acceptable locations of the water bearing wells. Then, we selected the following three classes of location:

- a. Location sited in correspondence of the major cities
- b. Location sited in selected fields, near the major cities keeping only few wells in cities,
- c. Location sited rather away from the urban and industrial centers, near the river.

The ratio  $w$  ( withdrawal / specific discharge of wells) depending on trasmissivity of the aquifers, increases as the distance from the main watercourses rises according to the Dupuit relation:

$$W = a \cdot \ln d \quad (11)$$

Where  $a$  is a coefficient depending on the trasmissivity and  $d$  is the distance from the rivers.

The groundwater pollution is very high in the wells located in cities; moreover the water quality is better in correspondence of the rivers, namely nearly river Ticino and river Adda.

Along watercourses in fact, large areas are characterized by the absence of groundwater pollution.

In the Milano province the probability to find groundwater polluted from contaminants of typology 4 or 5 nearly the major cities is higher than 30%.

This probability decreases approximatively according to the relation:

$$P = C \cdot e^{-d} \quad (12)$$

Where  $d$  is the distance (in %) from the town and  $C$  is the probability of pollution in the urban areas. The relations (10) and (11) demonstrate that the optimal location of the new wells should correspond to the areas located along the rivers.

We can easily point out many areas favorable to location of water wells ( namely near the river Ticino) able to yield a total discharge of about 1-2 mc.

However these areas can be utilized only with noticeable expenses due to the distance from the major cities. Hence, to solve the location problem, we synthesized these conditions in a diagram (Fig. 1) to demonstrate that optimal solution can be obtained under two conditions:

- a. The water bearing wells are located in areas where the sum of the decontamination and the withdrawal costs are minimized,
- b. Several water wells sited in correspondence of the major cities are utilized.

In fact the cost of wells in cities is largely acceptable, and the drawdown remains always very low. Moreover we calculated that the increase of about 3mc/s in water well discharge of Milano province could be reached without serious damage of the reserves and with acceptable costs.

All the other solutions cause major costs and damages to the water reserves.

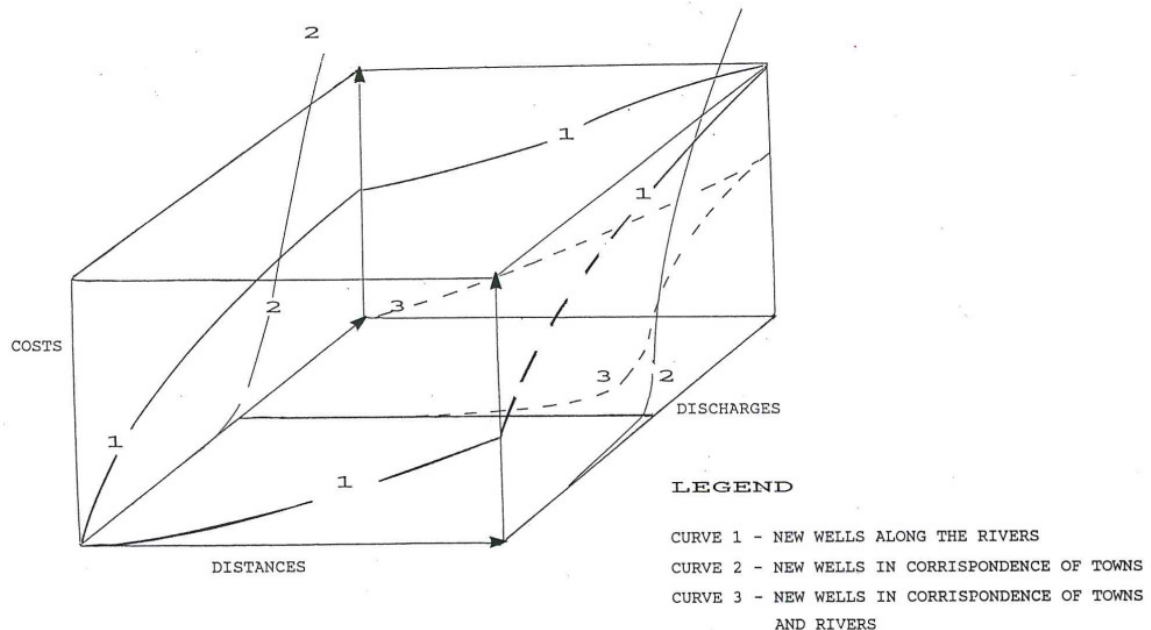


Fig. 1 - Curves Cost-Distances-Discharges corresponding to the main solutions.

## **CONCLUSION**

Further researches concerning the province of Como, Varese, Bergamo, Brescia and Cremona followed the hydrogeological studies on the Milano province illustrated in this paper.

These researches allowed to define for each area an optimal solution, different from these of other Provinces.

However, we can assert that the optimal solution ( i.e. with minor complexive cost) presents in general the following requirements:

- The most suitable location of new wells is always nearby the rivers;
- A noticeable contribution is required to superficial water ( namely to lakes);
- The aqueducts working at present must be maintained;
- Frequent control and monitoring of the water quality are required to maintain a good quality.

In conclusion, the analysis of the economical aspects of the planning and the management of the hydrogeological resources is based on the main concept that the groundwater must be considered as a good subject to trading rules. In fact, excessive wastes and uncontrolled consumptions of groundwater determine a strong economical impacts on the studied area, increasing the cost of urban , agricultural and industrial land uses.

On the other hand, this paper demonstrated that the most suitable solutions for a fairly management and preservation of the groundwater reserves can be founded by means of the above describes stages:

Stage a) study of the main hydrogeological features of the examined area and evaluation of the trend of the relative hydrogeological balance

Stage b) application of the Heidari's Model to select the areas containing the available reserves, able to assure the maximum drawdown in the critical points.

Stage c) evaluation of the costs of the selected solutions.

Stage d) choice of optimal location of the water bearing wells allowing the required yield, at minimum cost.

## **BIBLIOGRAPHY**

- Amministrazione provinciale di Milano et al., 1992 - **Presenza di composti organo alogenati nelle acque di falda della Provincia di Milano**. Ed. a cura della provincia di Milano.
- Beretta G.P., Cavallin A., Francani V., Mazzarella S. Pagotto A., 1985 – Primo bilancio idrogeologico della pianura milanese. *Acque sotterranee, Geograph, Mialno, 2,3,4: 1-34.*
- Heidari, M., 1982 - **Application of linear system's theory and linear programming to groundwater management in Kansas**. *Water Resources Bulletin, 18:1003-1012.*
- Maddock, T., 1972 - **Algebraic Technological Function from a Simulation Model**. *Water Resources Research, 8, 129-134.*
- Regione Lombardia, Sett. Ambiente ecologia, 1992 – Piano regionale di risanamento delle acque. Criteri di pianificazione in rapporto alla gestione delle risorse idriche lombarde. Ed. a cura della regione Lombardia.