AN APPLICATION OF STAGNATION POINTS THEORY

By V. Francani, L. Colombo, D. Cremonesi
vincenzo.francani@polimi.it, loris.colombo@polimi.it, daniele.cremonesi@polimi.it

Summary

1 INTRODUCTION

A problem often occurring in practice is the validation of efficacy of a barrier wells designed to containment of pollution. Several methods and solutions have been tested , both mathematical and experimental (i.e. employing togheter mathematical models and tracers). Here is presented the case of a barrier consisting of two wells aligned across to the motion field of a groundwater, with the same discharge Q (Fig. 1); the line connecting the pair of wells is normal to hydraulic gradient.

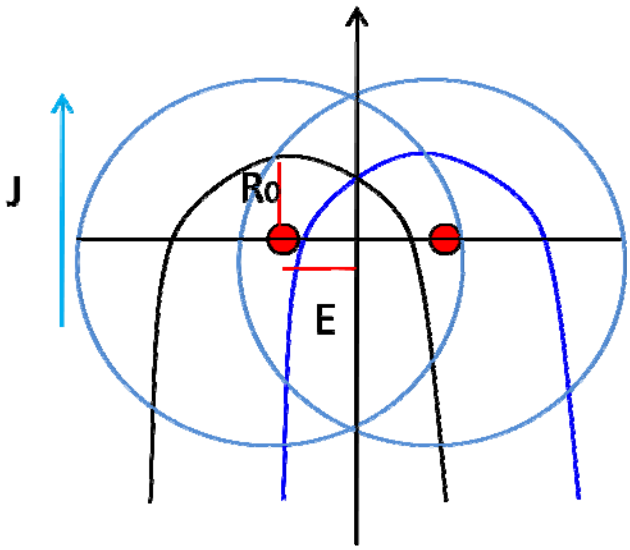

Fig.1 – A couple of wells giving rise to an elementary hydraulic barrier, and their composed capture zone.

In this case a piezometric composed depression grows up, with center line passing for the intersection point of the outline of the capture zone of each well.

Usually a test named “interference between wells test” is used to evaluate the efficiency of barriers: drawdown, pumping a flow Q by the tested well, are measured in the near wells, used as monitoring wells.
Data and measures obtained can be used if with traditional mehods (for example by means of step-drawdown test analysis) the parameters of the aquifer (such as transmissivity T) are evaluated, while the drawdown H – y, obtained with the pumping rate Q, are measured in piezometers and wells.
In the well – barrier close to the tested one the drawdown must be function of the pumping in the tested well, in order to evaluate the efficiency of the same well.
To evaluate the efficiency of the barrier, the piezometric level y must be measured in a piezometer at a distance x.
Referring to Fig. 2, it is important that long the line which joins the well – barrier points, the sum of velocity vectors j’ and j” towards to well A and B respectively, must be equal or higher the natural velocity of groundwater J.

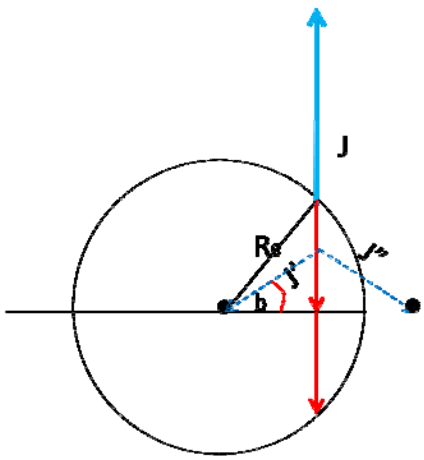

Fig. 2 – Hydraulic gradients J' e J" due to withdrawal and their sum (red vector)

It is important to verify the drawdown at the half distance of the wells: the superimposition is always obtained if the drawdown H – y is equal or superior to the one obtained in the case of the watershed are tangent. It is possible only if the distance 2E between wells is inferior to D (the capture curve is measurable with equation (1.1) in "Stagnation points: analytical – theoric approach and practical applications".

Data usually known are the flow Q in a well of the barrier, the y value measured in a piezometer set to a distance x from the well, the piezometric level h in stationary regime in a well and the gradient of groundwater. These data are used to evaluate the transmissivity T using the Dupuit's relationship. By these elements it is possible to simply evaluate also D, i.e. the width of the capture zone along the line joining the wells by means of the relationships:

$$D = \frac{Q}{2q} = Q/2TJ = \pi R_0/2 \qquad \textbf{(1)}$$

It is possible to evaluate if the two wells are closer enough to generate an efficient containment system, calculating D and comparing its value with d .

For suitability of the barrier, it needs that: d <D/2 **(2.1)**

so that:

d < Q/4Tj **(2.2)**

according with the solution of Javandel e al., highlighting that d optimal is 0,36 Ro.

It is also possible to verify the suitability of these criterion using measured piezometric data.

The y value measured in a piezometer set to a distance x from the well is used to evaluate the piezometric head y' at a distance x' equal to d (half distance between wells), using the Dupuit's relationship:

$$y' - y = \frac{Q}{2\pi T} \ln\left(\frac{x'}{x}\right) \qquad \textbf{(3)}$$

The suitability condition for the barrier is when drawdown H – y' is greather than drawdown obtained within a distance of D/2 from the well. The drawdown is determined first evaluating D/2 = Q/4TJ, then using Dupuit's relation adding to dynamic head h in the well, the value of Qln(x'/r)/πT.

Example of this method

If, by a discharge/drawdown test, T = 0.0004 m^2/s and Q = 0.001 m^3/s, J of the natural groundwater is 0.005, the distance between wells is 80 m and a piezometer located 20 m far from the tested well registers a drawdown equal to 25 cm. In the tested well, the piezometric level H is 4 m . Obviously the test demonstrates the overlap of the couple of capture zones, if the lowering of the piezometric head measured in monitoring piezometers along the symmetry axis , upstream of stagnation point , is greater than the drawdown calculated for the case of only tangent capture zone.

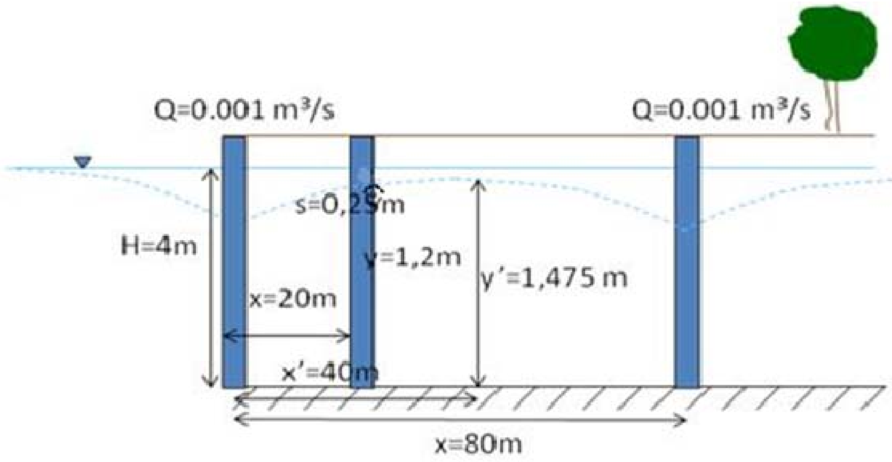

Fig. 2 – Illustration of the proposed example. In the figure are represented two well within a distance of 80 m and the calculation in the middle point y'

The semidistance E between wells is 40 m, and here y is obtained using (3): 1.475 m a.s.l.

Evaluating the piezometric level within a distance of E from both wells, corresponding it is observed that the elevation of piezometric level from the h value measured in well, is equal to half head elevation calculated with only one well.

So, when the barrier operates, y'= y_E value within a distance of E equal to 125 meters, is 1.92 m obtained using the following equation:

$$y_E - h = 0{,}5\frac{Q}{\pi T} ln\left(\frac{125}{r}\right)$$

(4)

The groundwater level calculated using the only pumping well in the medium distance is less than one evaluated along the capture curve of the two wells if all of them are pumping, with a double withdrawal, where the capture zones are only tangent (distance E from each well).

The point situated within a distance E from the test well is the medium one, the nearest to the test well respect to the all points between wells. After, following the flow direction, of this point, both capture curves and flux section towards the well are narrowed. Areas where piezometric depressions of each well sum are smaller following the flow direction, and became zero where capture curves intersecate. Downstream of the intersection points, there are no more points where waters run towards wells (as shown in Fig. 1).

2 A CASE STUDY

The stagnation point can be determined with described analytical methods. It is positioned where 0,5arctg (R0/E) = b, applying Pythagoras' Theorem to Fig. 3, where x is the hypotenuse and E and R0 the cathetus.

The solution can found out considering that $x = 2R_0/\sin b$ and $E^2+E^2tg^2b = x^2$

Then from :

$$x = 2R_0/\sin b \quad \text{and} \quad 1+tg^2 b = 1/\cos^2 b \tag{5}$$

derives that:

$$E^2/\cos^2 b = 4R_0^2/\sin^2 b, \text{ then } R_0/E = 0{,}5\tan b \tag{6}$$

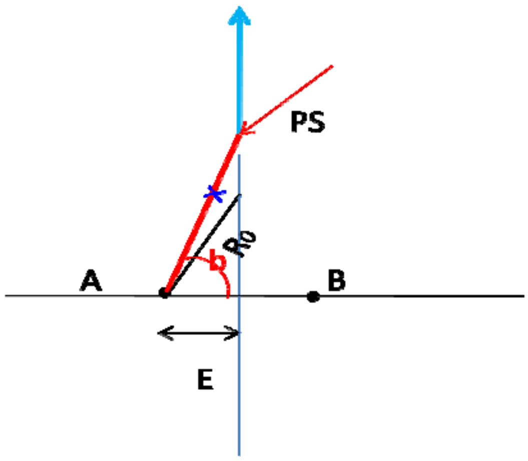

Fig. 3 – Location of the stagnation generated by two pumping wells

Therefore, knowing the b angle it's possible define the location of the stagnation point.

The availability of the location of the stagnation point is fundamental, because it limitates downstream the sector where piezometric depression of two wells contains the flux: along the axis beyond this point, drawdown is'nt able to recall waters toward the well.

3 REFERENCES

ANDERSON, MARY P. (1981), Seasonal reversals of groundwater flow around lakes and the relevance to stagnation points and lake budgets USA Water Resources Research 17, 1139

BEAR, J., 1979. Hydraulics of Groundwater. McGraw-Hill Book Co., New York.

CHRIST J.A, GOLTZ M.N., (2004), Containment of groundwater contamination plumes: minimizing drawdown by aligning capture wells parallel to regional flow, Journal of Hydrology 286, 52-68

CHRIST J.A, GOLTZ M.N., (2002),Hydraulic containment: analytical and semi-analytical models for capture zone curve delineation. Journal of Hydrology 262, 224-244

CHRIST J.A, GOLTZ M.N., HUANG J. (1999), Development and application of an analytical model to aid design and implementation of in situ remediation technologies. Journal of Contaminant Hydrology 37, 295-317

CHUNHUI LU, RULAN GONG, JIAN LUO (2009), Analysis of stagnation points for pumping well in recharge areas, Journal of Hydrology 373, 442-452MARY P. ANDERSON (1992), Long –and-short term transience in a groundwater/lake system in Wisconsin, USA Journal of Hydrology 145, 1-18